Superpositions and Categorical Quantum Reconstructions

Sean Tull

sean.tull@cs.ox.ac.uk
University of Oxford

SYCO 2
University of Strathclyde
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“Local and Global Phases in Categorical Quantum Theory”

The Plan

1. Motivation
2. Phased Biproducts
3. Relating Local and Global Phases
4. Quantum Reconstructions

1. Motivation

Two Categories for Quantum Theory

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Question 1: How is **Hilb** built from $\mathbf{Hilb}_{\mathsf{P}}$?

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A defining quantum feature are superpositions, corresponding to an addition operation $f + g$ in **Hilb**.

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This means that the κ_i form a coproduct of $\mathcal{H}, \mathcal{K}$:

$$\begin{array}{ccccc} \mathcal{H} & \xrightarrow{\kappa_1} & \mathcal{H} \oplus \mathcal{K} & \xleftarrow{\kappa_2} & \mathcal{K} \\ & {\scriptstyle f} \searrow & \downarrow {\scriptstyle \exists! h} & \swarrow {\scriptstyle g} & \\ & & \mathcal{L} & & \end{array}$$

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and the π_i dually form a product, in a compatible way.

However: $\mathcal{H} \oplus \mathcal{K}$ is not a biproduct in $\mathbf{Hilb}_{\mathrm{P}}$.

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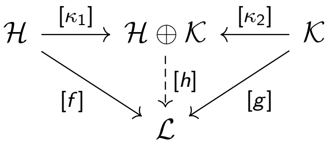

Commutes when $h \circ \kappa_1 = z \cdot f$ and $h \circ \kappa_2 = w \cdot g$ for global phases z, w.

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So $[h]$ exists but is now only unique up to a phase:

$$\mathcal{H} \oplus \mathcal{K} \xrightarrow{[U]} \mathcal{H} \oplus \mathcal{K} \qquad \text{with} \qquad U = \begin{pmatrix} \mathrm{id}_{\mathcal{H}} & 0 \\ 0 & z \cdot \mathrm{id}_{\mathcal{K}} \end{pmatrix}$$

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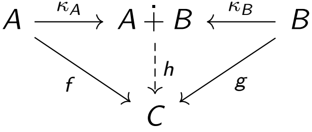

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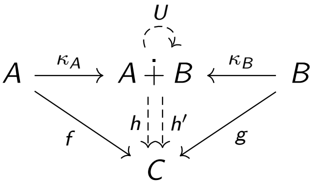

1. For all f, g as above there exists h making the diagram commute;
2. For any such h, h' we have $h' = h \circ U$ for some endomorphism U of $A \dot{+} B$ which is a phase, meaning that

$$U \circ \kappa_A = \kappa_A \qquad U \circ \kappa_B = \kappa_B$$

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Lemma

1. *They are unique up to (non-unique) isomorphism.*
2. *Any phase is an isomorphism.*
3. *Associativity holds:*

$$(A \dotplus B) \dotplus C \simeq A \dotplus B \dotplus C \simeq A \dotplus (B \dotplus C)$$

4. *Having finite phased coproducts* $A_1 \dotplus \cdots \dotplus A_n$
   $\iff$ *having binary ones* $A \dotplus B$ *and an initial object* 0.

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Can define phased products ($A \leftarrow A \dot{\times} B \rightarrow B$) dually, and even phased (co)limits more generally.

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with the same phases for each, and satisfying

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Example

Hilb$_\mathrm{P}$ has phased dagger biproducts given by the direct sum $\mathcal{H} \oplus \mathcal{K}$ of Hilbert spaces.

3. Relating Local and Global Phases

From Global to Local Phases

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Examples

$\mathbf{Hilb}_{\mathrm{P}}$ has phased biproducts as we've seen, arising from **Hilb** via the global phases $\mathbb{P} := \{z \in \mathbb{C} \mid |z| = 1\}$.
So does the quotient $\mathbf{Vec}_{\mathrm{P}}$ of $\mathbf{Vec}$:= k-vectors spaces and linear maps, via $\mathbb{P} := \{\lambda \in k \mid \lambda \neq 0\}$.

From Local to Global Phases

Observation:

$$\frac{\text{Linear maps } f : \mathcal{H} \to \mathcal{K}}{\Longleftrightarrow \text{Equivalence classes } \left[\begin{pmatrix} f & 0 \\ 0 & 1 \end{pmatrix}\right] : \mathcal{H} \oplus \mathbb{C} \to \mathcal{K} \oplus \mathbb{C}}$$

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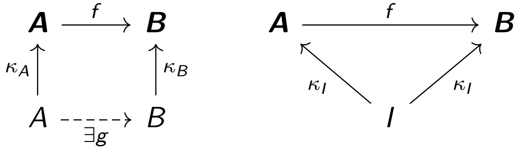
Definition

Let $(\mathbf{D}, \otimes)$ have phased coproducts. We define a category $\mathrm{GP}(\mathbf{D})$ by:

- objects are phased coproducts of the form $\boldsymbol{A} = A \dotplus I$ in $\mathbf{D}$;
- morphisms are those $f\colon \boldsymbol{A} \to \boldsymbol{B}$ in $\mathbf{D}$ with:



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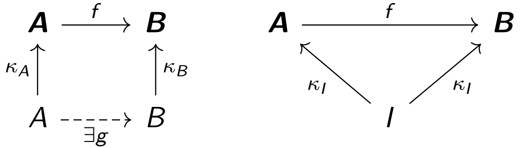

From Local to Global Phases

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Theorem

Let $\mathbf{D}$ *be a monoidal category with finite distributive phased biproducts (resp. 'nice' phased coproducts). Then* $\mathrm{GP}(\mathbf{D})$ *is a monoidal category with finite distributive biproducts (resp. coproducts) and a choice of global phases*

$$\mathbb{P} := \{u \colon \boldsymbol{I} \to \boldsymbol{I} \mid u \textit{ is a phase on } \boldsymbol{I} = I \dot{+} I \textit{ in } \mathbf{D}\}$$

such that

$$\mathbf{D} \simeq \mathrm{GP}(\mathbf{D})_{\mathbb{P}}$$

Summary

Biproducts and global phases		Phased Biproducts
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Remark

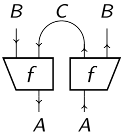
Results generalise beyond monoidal setting, to categories:

- **C** *with biproducts and trivial isomorphisms $A \simeq A$ on each object A*
- **D** *with phased biproducts and a phase generator I.*

4. Quantum Reconstructions

Generalised Quantum Theories

If **D** is a dagger compact category, then CPM(**D**) has the same objects and morphisms $A \to B$ being those in **D** of the form



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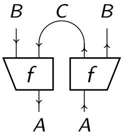

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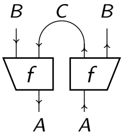

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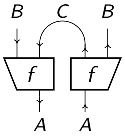

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Examples

$\mathbf{Quant}_{\mathbb{C}}$: fin. dim. Hilbert spaces and completely positive maps $f\colon B(\mathcal{H}) \to B(\mathcal{K})$.
$\mathbf{Quant}_{\mathbb{R}}$ is Quantum theory on real Hilbert spaces.

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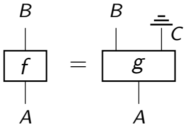
- *dagger compact subcategory* $\mathbf{C}_{\mathrm{pure}}$ *satisfying purification:*

$\forall f$  *for some* $g \in \mathbf{D}_{\mathrm{pure}}$

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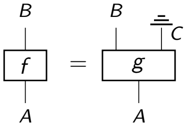

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We will say that $\mathbf{C}_{\mathrm{pure}}$ has the superposition properties when it has finite phased dagger biproducts satisfying some mild conditions.

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Theorem

Let $(\mathbf{C}, \mathbf{C}_{\mathsf{pure}}, \ground)$ be an environment structure for which $\mathbf{C}_{\mathsf{pure}}$ has the superposition properties. Then there is an embedding

$$\mathbf{Quant}_S \hookrightarrow \mathbf{C}$$

preserving $\dagger, \otimes, \ground$, for some involutive semi-ring S with $\mathbf{C}_{\mathsf{pure}}(I, I) \simeq S^{\mathrm{pos}}$.

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preserving $\dagger, \otimes, \bar{\top}$*, for some involutive semi-ring* S *with* $\mathbf{C}_{\text{pure}}(I, I) \simeq S^{\text{pos}}$*.*

Proof.

$\text{GP}(\mathbf{C}_{\text{pure}})$ has biproducts, so contains $\mathbf{Mat}_S$ for its scalars S. Then

$$\mathbf{Quant}_S \hookrightarrow \text{CPM}(\text{GP}(\mathbf{C}_{\text{pure}})) \simeq_{\star} \text{CPM}(\mathbf{C}_{\text{pure}}) \simeq \mathbf{C}$$

where $\star$ follows from our assumptions on $\mathbf{C}_{\text{pure}}$. □

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Thanks for listening!

References

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